THEORY OF GRAVITATIONAL STABILITY OF A ROTATING CYLINDER

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The stability of a rotating dust cylinder against perturbations located in the plane perpendicular to the axis of rotation is investigated. It is shown that a homogeneous rotating cylinder containing a weak inhomogeneity is stable against such perturbations. A weakly inhomogeneous cylinder with opposite streams of equal density is unstable for the $l = 2$ mode in the case of a perturbation of the form $\sim e^{i(l\varphi - \omega t)}$, when the density increases radially. The instability of a system consisting of a homogeneous rotating dust cylinder in a hot homogeneous medium is determined. It is shown that the maximum growth rate corresponds to $l = 2$ when the density of a cold cylinder is not negligible in comparison with the density of the medium. In the opposite case, the maximum growth rate shifts toward $l = 3$. An attempt is made to associate the existence of the maximum growth rate for $l = 2$ with the presence of two spiral arms in most galaxies. It is shown that, when the longitudinal temperature is high enough, a rotating cylinder which is bounded in the radial direction is stable against arbitrary perturbations.

The gravitational instability of a homogeneous medium was first investigated by Jeans [1]. It was subsequently noted [2-3] that Jeans' analysis was not wholly correct because there was no equilibrium in a homogeneous gravitating medium. A correct analysis of the homogeneous medium, with account for time dependence, was also given in [2, 3]. The results turned out to be close to those obtained by Jeans, namely, that a homogeneous medium supports growing perturbations with wavelength in excess of the critical value

$$\lambda > \lambda_* = 8\pi c / \sqrt{4\pi G\rho}$$

where ρ is the density, c is the speed of sound, and G is the gravitational constant.

The unperturbed solution is time-dependent and the perturbations do not grow exponentially but in accordance with a more complicated law which is approximately of the form $\exp(\int \omega(t)\, dt)$. The unperturbed system can be taken as the equilibrium (time-independent) system because of its finite size and the pressure gradient which balances the gravitation. However, in the simplest case, it then turns out that the size of gravitating bodies in equilibrium is on the order of the critical wavelength, so that Jeans instability does not occur.

The medium whose equilibrium and stability is considered may be a gas with a given equation of state and short mean free path due to nongravitational interaction of atoms, ions, and electrons.

However, if the interaction between the particles in the system is purely gravitational, the two-body force in an equilibrium system is, roughly speaking, weaker by a factor of N (or, more precisely, N/ln N) than the collective interaction. Consequently, the problem reduces in first approximation to the determination of particle motion in the collective self-consistent gravitational field (this is not valid if there are direct inelastic stellar encounters; henceforth we confine our attention to systems in which inelastic collisions are sufficiently rare to be negligible). Such collisionless motion is described by the Boltzmann-Vlasov kinetic equation [4, 14]. A similar situation is characteristic, as an example, for stars in our own Galaxy and certain other similar galaxies.

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Correspondingly, perturbations of the distribution function, i.e., of the density in velocity and coordinate space, must be considered in stability problems.

Stability depends on the initial distribution function. If the initial distribution is Maxwellian, the final results are not very different from the hydrodynamic case [5-9, 11].

The gravitational instability obtained in the above papers is connected with the critical Jeans wavelength and appears only in the absence of an equilibrium state.

Since the gravitational and Coulomb forces are both dependent on interparticle distance, one would expect the appearance in the gravitating medium of instabilities analogous to the kinetic plasma instabilities, namely, the two-stream instability and the anisotropic instability.

In fact, for a non-Maxwellian distribution function these instabilities do appear in the equilibrium state: the anisotropic instability is obtained for an anisotropic Maxwellian distribution [12] and the two-stream instability [4] occurs if there are "humps" on the one-dimensional velocity distribution function $f(v)$, i.e., if there are regions with $\partial f / \partial v > 0$, $v > 0$.

Equilibrium of a gravitating medium in a homogeneous state is impossible: the necessary condition for equilibrium is inhomogeneity or anisotropy.

This may lead to the suppression of the two-stream instability in the equilibrium state in the presence of two opposite beams which in plasma are always unstable (see section 4). Therefore, the criteria for two-stream instability which have been established for plasma cannot be directly generalized to a gravitating medium.

The instability of a number of exact solutions for a rotating equilibrium cylinder is investigated below, along with the stability of a rotating dust cylinder against perturbations in the plane perpendicular to the axis of rotation. For a dust medium, the kinetic analysis is fully equivalent to the hydrodynamic approach. In a solution with oppositely rotating streams there is two-stream instability in the presence of an inhomogeneity when the density increases with distance from the axis of rotation.

We also show that the rotation of a dust cylinder in a hot stationary medium exhibits the two-stream instability (the perturbations are taken to be of the form $\sim\exp[i(l\varphi - \omega t)]$ throughout) for which the maximum growth rate occurs at $l = 2$, as in the preceding case of an inhomogeneous rotating cylinder with opposite beams. This is used as a basis for a discussion of the observed presence of two arms in most spiral galaxies (see section 6 on the work of Lin, Marochnik, and others).

It is shown that a rotating, infinitely long cylinder, bounded in the radial direction, is stable against perturbations if the longitudinal temperature $T_\parallel$ is such that $T_0 < T_\parallel$.

1. Homogeneous Cylinder. Consider a dust cylinder (pressure $P = 0$) which is in equilibrium when the gravitational force is balanced by the centrifugal rotation force. The problem is assumed to be two-dimensional (although gravitation is three-dimensional) and we consider only the dependence on r and φ in a cylindrical set of coordinates. In the stationary state

$$v_{r0} = 0, \qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{d\Phi_0}{dr}\right) = 4\pi G\rho_0, \text{ and } \quad \frac{v_{\varphi 0}^2}{r} = \frac{d\Phi_0}{dr} \tag{1.1}$$

where v_r and v_φ are the velocities in the radial and cross-radial direction and Φ is the gravitational potential. For a homogeneous cylinder

$$v_{\varphi 0}{}^2 = 2\pi G\rho_0 r^2 \tag{1.2}$$

We investigate the stability of the stationary state against small perturbations. The perturbed solution, which is not very different from the stationary solution, is sought in the form

$$v_r = V_r(r)\, e^{i(l\varphi - \omega t)}, \qquad v_\varphi = V_\varphi(r)\, e^{i(l\varphi - \omega t)} \tag{1.3}$$

The linearized equations for the motion of dust on a plane are of the form

$$-i\left(\omega - l\frac{v_{\varphi 0}}{r}\right)v_r - 2\frac{v_{\varphi 0}}{r}v_\varphi = -\frac{d\Phi}{dr}$$

$$\left(\frac{v_{\varphi 0}}{r}+\frac{dv_{\varphi 0}}{dr}\right)v_r - i\left(\omega - l\frac{v_{\varphi 0}}{r}\right)v_\varphi = -i\frac{l}{r}\Phi$$
$$\left(\frac{\rho_0}{r}+\frac{d\rho_0}{dr}\right)v_r + i\rho_0\frac{l}{r}v_\varphi - i\left(\omega - l\frac{v_{\varphi 0}}{r}\right)\rho + \rho_0\frac{dv_r}{dr} = 0$$
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right) - \frac{l^2}{r^2}\Phi = 4\pi G\rho \tag{1.4}$$

where we have taken into account Eq. (1.3).

In the homogeneous case, $r^{-1}v_{\varphi 0} = \Omega = \text{const}$ and the first three equations reduce to

$$\frac{\rho}{\rho_0}[4\Omega^2 - (\omega - l\Omega)^2] = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right) - \frac{l^2}{r^2}\Phi \tag{1.5}$$

If we compare this with the last equation in Eq. (1.4), we find that the set given by Eq. (1.4) has a solution subject to a condition which can be reduced to the following dispersion relation:

$$4\Omega^2 - (\omega - l\Omega)^2 = \omega_0^2, \quad \Omega^2 = 2\pi G\rho_0$$
$$\omega = l\Omega \pm \omega_0, \quad \omega_0^2 = 4\pi G\rho_0 \tag{1.6}$$

Therefore, the stationary state of a homogeneous rotating cylinder is stable against perturbations of the above kind, and the resulting dispersion relation is the analog of the corresponding equation for plasma oscillations (but only in a rotating frame of reference).

2. Inhomogeneous Cylinder. Let us now determine the eigenfunctions for plasma-type oscillations, as specified by Eq. (1.6), in the more general case of an inhomogeneous cylinder.

Equation (1.4), which describes small oscillations of a rotating inhomogeneous cylinder, can be reduced to the following second-order differential equation:

$$\left(1+\frac{y^2}{\omega_0^2}\right)\Delta\Phi + A_1\Phi' + \frac{2l\Omega}{rx}\left(A_1+\frac{\Omega'}{\Omega}\right)\Phi = 0 \tag{2.1}$$

where

$$y^2 = x^2 - 2\alpha\Omega, \quad \alpha = 2\Omega + r\Omega', \quad x = l\Omega - \omega,$$
$$k_0 = \frac{1}{\rho_0}\frac{d\rho_0}{dr}, \quad A_1 = k_0 + 2\,\frac{\Omega(\alpha + \alpha'\Omega/\Omega' - lx)}{x^2 - 2\alpha\Omega}\,\frac{\Omega'}{\Omega} \tag{2.2}$$

Equation (2.1) is most simply solved in the WKB approximation in the case of weak inhomogeneity. Expanding the density ρ_0 around the constant density ρ_c, we obtain

$$\Omega = \Omega_c + \frac{2\pi G\rho_c - \Omega_c}{r_c}(r - r_c), \quad \Omega_c^2 = \int_0^{r_c}\frac{4\pi G\rho r\,dr}{r_c^2}$$
$$\mu = 2\pi\int_0^{r_c}\rho r\,dr, \quad \bar\rho = \frac{\mu}{\pi r_c^2}, \quad \Omega_c^2 = 2\pi G\bar\rho \tag{2.3}$$

Substituting this into Eq. (2.1) with $|kr| \gg 1$ (WKB approximation), we obtain a stable solution. When $l = 0$, the cylinder is stable for an arbitrary inhomogeneity because the angular momentum of each particle is conserved [15].

When Eq. (2.1) is analyzed, we must first verify that the solution is, in fact, an eigenfunction of the equation. Equation (2.1) has poles at the points

$$r = 0, \quad \omega_0^2(r) + y^2(r) = 0, \quad x(r) = 0, \quad y^2(r) = 0$$

Singularities in the equation may ensure that its solutions include singular solutions (see, for example, [16]). However, all the physical quantities must be finite, and hence the proof that the solution of the equation is an eigenfunction reduces to the demonstration that the solution is finite at any point r.

Consider the case of small departures from inhomogeneity

$$[\rho(R) - \rho(0)]/\rho(r) \ll 1$$

In this case, Eq. (2.1) becomes

$$\Phi'' = \left(\frac{1}{r} + \alpha_1\right)\Phi' - \left(\frac{l^2}{r^2} + \frac{\beta}{r}\right)\Phi = 0 \quad \left(\frac{d\Omega}{dr} = \mathrm{const}\right) \tag{2.4}$$

where α_1 and β are constants related to k and ω.

The solution of Eq. (2.4) can be expressed in terms of the Whittaker function [17] as follows:

$$\Phi = r^{-1/2} e^{-1/2\alpha_1 r} W_{\lambda,l}(\alpha_1 r), \quad \lambda = -\frac{\alpha_1 + 2\beta}{2\alpha_1} \tag{2.5}$$

The solution of Eq. (2.4) given by Eq. (2.5), which is bounded for $r \to 0$ and $r \to \infty$, exists for $\lambda - k - 1/2 = n$, where n + 1 is a natural number [17], and this determines the real values of ω. This solution decreases exponentially for $r \to \infty$ and $\sim r^l$ as $r \to 0$, $l \geq 1$. The solution of Eq. (2.1) is therefore finite.

3. Rotating Cylinder with Opposite Beams. Let us now consider a homogeneous dust cylinder consisting of two mutually penetrating beams with velocities v_{φ_0} and $-v_{\varphi_0}$ and equal density $1/2\rho_0$, so that we can investigate the analog of the two-stream plasma instability. The angular momentum of the cylinder is zero (in contrast to the above homogeneous rotation), and Eqs. (1.1) and (1.2) are valid in the stationary state. The dispersion relation for this case can be obtained by writing Eq. (1.5) for each of the beams with density $1/2\rho_0$, and then comparing it with the last equation in Eq. (1.4) for the perturbed potential. The result is

$$\frac{1}{4\Omega^2 - (\omega - l\Omega)^2} + \frac{1}{4\Omega^2 - (\omega + l\Omega)^2} = \frac{1}{\Omega^2} \tag{3.1}$$

For ω^2 we then have

$$\omega^2 = 1/2\,\omega_0^2\left(l^2 + 3 \pm 2\sqrt{3l^2 + 1/4}\right) \tag{3.2}$$

The quantity ω^2 has a minimum at $l = 2$ for which $\omega^2 = 0$, i.e., we have unconditional equilibrium. Moreover, $\omega^2 > 0$ when $l \neq 2$. If the density of the two interpenetrating beams is different, the resulting dispersion relation is more complicated, but it can be shown that all its roots are real. It follows that the case which we are considering is different from the plasma case, in which the opposite beams are always unstable.

When the inhomogeneous cylinder consists of two identical but opposite rotating beams, then instead of Eq. (2.1) we have

$$2\frac{\Delta\Phi}{\omega_0^2} = K_+ + K_-, \quad K_+ = -\frac{1}{y^2}\left[\Delta\Phi + A_1\Phi' + \frac{2l\Omega}{rx}\left(A_1 + \frac{\Omega'}{\Omega}\right)\Phi\right] \tag{3.3}$$

where K_- is obtained from K_+ by replacing Ω with $-\Omega$ throughout Eq. (2.2). When $l/kr \ll 1$, we have from Eq. (3.3)

$$\frac{1}{2\alpha\Omega - (\omega - l\Omega)^2} + \frac{1}{2\alpha\Omega - (\omega + l\Omega)^2} = \frac{2}{\omega_0^2} \tag{3.4}$$

It is readily verified that Eq. (3.4) becomes identical with Eq. (3.1) when $\rho_0' = \Omega' = 0$. Using Eq. (2.2) and Eq. (3.4), we obtain the dispersion relation in the form

$$\omega^4 - 2\omega^2\left[(4 + l^2)\Omega^2 - 1/2\,\omega_0^2 + 2r\Omega\Omega'\right] + \left[(4 - l^2)\Omega^2 + 2\Omega\Omega' r\right]^2 - \omega_0^2\left[(4 - l^2)\Omega^2 + 2\Omega\Omega' r\right] = 0 \tag{3.5}$$

Equation (3.5) is biquadratic in ω and its determinant is positive so that the instability described by this equation can only be aperiodic, i.e., the growth rate is $\gamma = i\omega$. Our analysis of the instability of a homogeneous cylinder with opposite beams shows that the maximum growth rate in the case of the inhomogeneous cylinder must be sought for $l = 2$. In fact, when the density gradient is small, the instability can occur only for $l = 2$:

$$\omega^2 = -r\Omega\Omega' \frac{\omega_0^2}{8\Omega^2 - {}^1/_2\omega_0^2} \tag{3.6}$$

$$8\Omega^2 > {}^1/_2\omega_0^2 \qquad \text{for small } \rho_0' \text{and } \Omega'$$

It is clear from these expressions that the necessary condition for the onset of the two-stream instability in the above cases is that the angular velocity increase with distance from the center of the cylinder. The equilibrium states of the gravitating dust cylinder in which the particles move on noncircular orbits is described in [18].

4. Instabilities of a Rotating Dust Cylinder in a Hot Medium. Let us investigate the spectrum of oscillations produced when a homogeneous gravitating dust cylinder is placed in a hot medium. It was shown above that the characteristic frequencies of the rotating homogeneous cylinder are real. In the hot medium these oscillations may grow as a result of a resonant interaction of the wave with the medium particles. The linearized kinetic equation for the hot gas is

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \nabla\Phi\frac{\partial f_0}{\partial \mathbf{v}} = 0 \tag{4.1}$$

where the term $\nabla\Phi_0\ \partial f/\partial \mathrm{v}$ is neglected because $\nabla\Phi_0\ \partial f/\partial \mathrm{v} \ll \mathrm{v}\partial f/\partial \mathrm{r}$, which can be written in the form

$$kr\frac{\Omega^2}{k^2 v_T^2} \ll 1, \quad v_T^2 = \frac{2T}{m} \tag{4.2}$$

Substituting the perturbed distribution function in the form

$$f = Fe^{i(\mathbf{kr}-\omega t)}$$

we obtain the perturbed density of the hot medium in the form

$$\rho^* = -im\nabla\Phi\int\frac{\partial f_0/\partial \mathbf{v} d^2v}{\omega - \mathbf{kv}} \tag{4.3}$$

where m is the mass of a particle of the hot medium, which is assumed to consist of identical particles.

Using Eqs. (1.4), (1.5), and (4.3), we find that the linearized Poisson equation is of the form

$$\frac{d^2\Phi}{dr^2} + \frac{A}{r}\frac{d\Phi}{dr} - \frac{l^2}{r^2}B\Phi = 0 \tag{4.4}$$

$$A = 1 - \frac{4\pi Gimr}{1-\omega_0^2\Delta}J_r, \quad B = 1 - \frac{4\pi Grm}{(1-\omega_0^2\Delta)l}J_\varphi$$

$$J_r = \int\frac{(\partial f_0/\partial v_r)d^2v}{\omega - kv_r - lv_\varphi/r}, \quad J_\varphi = \int\frac{(\partial f_0/\partial v_\varphi)d^2v}{\omega - kv_r - lv_\varphi/r} \tag{4.5}$$

$$f_0 = \frac{n_0}{\pi v_T^2}\exp\left(-\frac{v_r^2+v_\varphi^2}{v_T^2}\right), \quad \Delta = \frac{1}{4\Omega^2-(\omega-l\Omega)^2}$$

Using the residue theorem [19, 20], we obtain the following expressions for the integrals in Eq. (4.5):

$$J_r = \begin{cases}\vartheta(1+i\sqrt{\pi}\,\theta), & l/kr \ll 1\\ i\vartheta\sqrt{\pi}\,\theta kr/l, & l/kr \gg 1\end{cases} \quad \left(\vartheta = \frac{2n_0}{kv_T^2}\right) \tag{4.6}$$

$$J_\varphi = \begin{cases}i\vartheta\sqrt{\pi}\,\theta kr/l, & l/kr \ll 1\\ \vartheta(1+i\sqrt{\pi}kr\,\theta/l)\,kr/l, & l/kr \gg 1\end{cases} \quad \left(\theta = \frac{\omega}{kv_T}\right) \tag{4.7}$$

Consider the case $l/\mathrm{kr} \ll 1$

$$A = 1 - \frac{2ikr}{1-\omega_0^2\Delta}\left(1 + \frac{i\sqrt{\pi}\,\omega}{kv_T}\right)\frac{\omega_0^{*2}}{k^2v_T^2} \tag{4.8}$$

$$B = 1 - \frac{2i\sqrt{\pi}\,k^2r^2}{1-\omega_0^2\Delta}\frac{\omega\omega_0^{*2}}{k^3l^2v_T^3} \qquad (\omega_0^{*2} = 4\pi G\rho_0^*) \tag{4.9}$$

The frequency can be written in the form

$$\omega = \omega_1 + i\gamma \tag{4.10}$$

In that case, $\gamma \ll \omega_1$. From Eqs. (4.4) and (4.9), neglecting the unity in the expression for A, i.e., using the inequality

$$kr \gg kv_T / \Omega \tag{4.11}$$

we then have the following expression:

$$\omega_1 = \omega_0\left[\frac{l}{\sqrt{2}}\left(1 + \frac{\omega_0^{*2}}{\omega_0^2}\right)^{1/2} \pm \left(1 + 2\frac{\omega_0^{*2}}{\omega_0^2}\right)^{1/2}\right] \tag{4.12}$$

The inequalities given by Eqs. (4.11) and (4.12) are consistent for $kr \gg 1$, provided that

$$\Omega / kv_T \ll 1 \tag{4.13}$$

This inequality gives the lower limit for the temperature of the hot medium for fixed k and Ω.

Equation (4.12) is consistent with the inequality given by Eq. (4.13), and in deriving it we have used the following expression for Ω, which is a consequence of the condition of equilibrium:

$$\Omega^2 = \frac{1}{2}(\omega_0^2 + \omega_0^{*2}) \tag{4.14}$$

Since $\gamma \ll \omega_1$, the numerator in the coefficients A and B of Eqs. (4.8) and (4.9) is approximately equal to

$$1 - \frac{\omega_0^2}{4\Omega^2 - (\omega - l\Omega)^2} = -\frac{2i\gamma(\omega_1 - l\Omega)}{\omega_0^2} + 1 - \frac{\omega_0^2}{4\Omega^2 - (\omega_1 - l\Omega)^2} \tag{4.15}$$

Substituting this into Eqs. (4.4)-(4.9), we obtain

$$\gamma_{1,2} = -2\sqrt{\pi}\,\frac{\omega_0^{*2}\omega_0^2}{k^3 v_T^3}\left[1 \pm \frac{l}{\sqrt{2}}\left(\frac{\omega_0^2 + \omega_0^{*2}}{\omega_0^2 + 2\omega_0^{*2}}\right)^{1/2}\right] \tag{4.16}$$

When $l = 0.1$, this expression gives the damping rate $\gamma_{1,2} < 0$. When $l \geq 2$, for the case in which in the parentheses in Eq. (4.16) we take the negative sign, we obtain $\gamma_2 > 0$; and these are growing oscillations due to the resonant interaction of waves with the particles of the hot medium (two-stream instability).

It is clear from Eq. (4.16) that the quantity $\gamma \sim l/(kr)^3$ is largely determined by the parameter kr and reaches a maximum for $kr \to 1$. Since in this case l/kr is a small parameter of the problem, the maximum of the growth rate is reached for minimum l, i.e., for $l = 2$. For $\omega_0^* \gg \omega_0$ the maximum growth rate shifts toward $l = 3$.

Suppose now that $l/kr \gg 1$ so that, using Eqs. (4.6) and (4.7) and proceeding by analogy with the foregoing, we obtain

$$\gamma_{1,2} = -2\sqrt{\pi}\left(\frac{kr}{l}\right)^3 \frac{\omega_0^{*2}\omega_0^3}{k^3 v_T^3}\left[1 \pm \frac{l}{\sqrt{2}}\left(\frac{\omega_0^2 + \omega_0^{*2}}{\omega_0^2 + 2\omega_0^{*2}}\right)^{1/2}\right] \tag{4.17}$$

The physical meaning of the instability obtained in this section is illustrated in Fig. 1. The velocity distribution function for the dust particles in the cylinder takes the form of a "hump" on the distribution function for the particles of the hot medium. This explains the presence of the two-stream instability due to the resonant Landau mechanism (for further details see, for example, [20]).

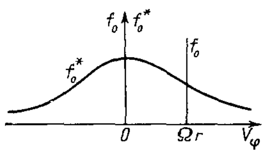

Fig. 1

5. Stability of A Rotating Cylinder Bounded in the Radial Direction. Consider a homogeneous rotating cylinder of infinite length, but bounded in the radial direction, in which the centrifugal force balances the gravitational force in a plane.

A cylinder of this kind is in equilibrium, but in the dust this equilibrium is unstable against perturbations with $k_z \neq 0$, and for small values of k_z the square of the frequency is $\omega_d^2 \sim -(k_z R)^2 \omega_0^2$, where R is the radius of the cylinder and $\omega_0^2 = 4\pi G\rho_0$. For large k_z we have, as in the case of the homogeneous medium,

$\omega^2 = -\omega_0^2$. If there is a nonzero longitudinal temperature $T_\parallel$ we have stabilization, and dimensional considerations show that for any k_z the change in the square of the frequency due to the appearance of $T_\parallel$ is $\Delta\omega^2 \sim k_z^2 v_T^2$.

Perturbations with sufficiently large k_z will be stabilized for any $T_\parallel$, but since ω_d^2 and $\Delta\omega^2$ have the same dependence on k_z for small k_z and for $T_\parallel > T_0$, the perturbations with small k_z will also be stabilized.

Therefore, the cylinder is completely stable for $T_\parallel$ such that $v_T > v_{T_0} \sim R\omega_0$.

Let us prove this rigorously and determine the precise value of T_0. It is shown in [12] that, when $T_\parallel > T_\perp$, the anisotropic instability is absent, $T_\perp = 0$ and, therefore, the cylinder is also kinetically stable.

In the case of anisotropic pressure we cannot write the equation of state in the form $P = P(\rho)$ just as in the case of a cylinder with longitudinal temperature.

Let us make use of the equations for the longitudinal and perpendicular components of the pressure tensor $P_\parallel$ and $P_\perp$ [21]. From the linearized Euler equations, the continuity and gravitational equations, and the equations for $P_\parallel$ and $P_\perp$, we obtain the following equation for the perturbation of the potential Φ:

$$\Phi'' + \frac{\Phi'}{r} - \Phi\left(\frac{l^2}{r^2} + \nu\right) = 0, \quad r < R \tag{5.1}$$

$$\Phi'' + \frac{\Phi'}{r} - \Phi\left(\frac{l^2}{r^2} + k^2\right) = 0, \quad r < R$$

$$\nu = \frac{k^2(4\Omega^2 - x^2)(x^2 - k^2c^2 + \omega_0^2)}{(x^2 - k^2c^2)(4\Omega^2 - x^2) - \omega_0^2(x^2 - {}^2/_3 k^2c^2)}$$

$$c^2 = \frac{3p_0}{\rho_0} = 3\,\frac{kT_\parallel}{m}, \quad \Phi \sim \exp(l\varphi + kz - \omega t)$$

The solution of this equation must be finite throughout, must vanish at infinity, and both the solution and its first derivative must be continuous.

Solutions satisfying these conditions are

$$\Phi = AJ_l(qr), \quad r < R, \quad \nu = -q^2 BK_l(kr), \quad r > R, \tag{5.2}$$

where J_l is the Bessel functions of order l, and K_l is the MacDonald function [22]. Since the solution of Eq. (5.2) and its first derivative must be continuous for $r = R$, we have

$$AJ_l(qR) - BK_l(kR) = 0$$

$$Aq\left[J_{l-1}(qR) - \frac{l}{qR}J_l(qR)\right] + Bk\left[K_{l-1}(kR) + \frac{l}{kR}K_l(kR)\right] = 0 \tag{5.3}$$

The condition for a nontrivial solution yields the following dispersion relation:

$$\frac{qJ_{l-1}(qR)}{J_l(qR)} = -\frac{kK_{l-1}(kR)}{K_l(kR)} \tag{5.4}$$

where $q(k, \omega)$ is defined by Eqs. (5.1) and (5.2). It follows from Eq. (5.4) that q has a minimum $q_{min} > 0$, and $q_{min} R = 2.4$ is the first zero of $J_0(x)$. Let us now find $\omega(k, q)$. From Eqs. (5.1) and (5.2) we have

$$x^2 = (\omega - l\Omega)^2 = \frac{\omega_0^2 + k^2c^2}{2} \pm \left[\frac{(3\omega_0^2 - k^2c^2)^2}{4} - \frac{2}{3}\,\omega_0^2(3\omega_0^2 - k^2c^2)\frac{q^2}{k^2 + q^2}\right]^{1/2} \tag{5.5}$$

from which it follows that

$$x^2 > 0 \quad \text{for} \quad c^2 > \frac{\omega_0^2}{k^2 + {}^2/_3 q^2} \tag{5.6}$$

When $c^2 > {}^3/_2\,\omega_0^2/q_{min}^2$ the cylinder is stable for all k and l. Using the expression for c^2 and q_{min}, we find that a cylinder bounded in the radial direction is stable against any perturbations, provided

$$\frac{T_\parallel}{m} > 0.087\,\omega_0^2 R^2$$

6. Conclusion. The main results of this paper are (1) the conclusion that a rotating cylinder is stable against arbitrary perturbations in the plane perpendicular to the axis of rotation, and (2) the demonstration

of the kinetic two-stream instability of a rotating cylinder in a hot gas, which has a maximum growth rate for $l = 2$, where $\omega_0^{*2} \gg \omega_0^2$.

This instability can be associated with the presence of two spiral arms in most galaxies [23]. These arms may be a consequence of the instability produced during the rotation of the gas, which can be discussed hydrodynamically against a background of collisionless stars. The formation of the arms was discussed in [24] as a consequence of gravitational instability. The effect of kinetic instability on the formation of spiral arms in a disk was discussed in [25].

The infinite-cylinder model discussed here is unrelated to a galaxy in the form of a highly oblate ellipsoid. However, it is quite possible that the presence of the maximum growth rate for $l = 2$ in the case of the two-stream instability will also be confirmed for more complicated configurations, as compared with the infinite cylinder.

LITERATURE CITED

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